Brief Practical Guide

to the

Scanning Tables

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(November 1996, Revised September 2003)

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If a crystal of a given space group symmetry is transected by a plane of a crystallographic orientation, i.e. an orientation given by integer Miller or Miller-Bravais indices, the subgroup of all elements of the space group which leaves the plane invariant is a layer group. The layer groups corresponding to all planes of a crystallographic orientation for all space groups are tabulated in the scanning tables.

The tables are given according to the 230 space groups. The order of the space groups follows that given in Volume A of the *International Tables for Crystallography* (*ITC(1983)*). In general, there is one table for each of the 230 space groups. If in *ITC(1983)* there are two tabulations given for a specific space group, corresponding to two choices of origin, then two tables are given. If multiple cell choices are given in *ITC(1983)*, then multiple tables are given.

Explicit tables are given for all triclinic and monoclinic space groups. For all other space groups, explicit tables are given only for orientations of planes with fixed values of Miller or Miller-Bravais indices. For these other space groups and orientations of planes with variable values of Miller or Miller-Bravais indices auxiliary tables are given from which explicit tables can be constructed.

We describe the format and content of the explicit tables, first for orientations with fixed values of indices, and second for orientations with variable values of indices. We then show how to use the auxiliary tables to construct tables for the remaining cases.

Explicit Scanning Tables

The content and arrangement of the explicit scanning tables is as follows:

1. Headline
2. Orientation orbit
3. Conventional basis of the scanning group
4. Scanning group
5. Linear orbit
6. Sectional layer group

1. Headline

The first line of the headline begins with the serial number of the space group type which follows the numbering given in *ITC(1983)*. This is followed by the short Hermann Mauguin symbol and the Schonflies symbol for the space group type.

The second line gives the full Hermann Mauguin symbol of the specific space group, of the type listed in the first line, which is used in the table. This is followed by a statement of origin in those cases where two space groups of different origin are considered, or by a statement of cell choice when space groups of different cell choices are considered.

The specific space groups considered, including their orientation and choice of origin are those tabulated in *ITC(1983)*. These specific space groups are defined in *ITC(1983)* by a diagram, the given *symmetry operations*, or by the coordinates of the given set of *general positions*. The diagram is that given in *ITC(1983)* with the upper-left-hand corner of the diagram taken as the origin

P , its left edge downwards as the vector **a**, its upper edge to the right the vector **b**, and the vector **c** up out of the page.

2. Orientation Orbit

The first column in each table is titled *Orientation Orbit (hkl)* or *Orientation Orbit (hkil)*. In this column are listed the Miller indices or the Miller-Bravais indices of the planes under consideration.

Sets of planes have orientations which are related by rotations and rotation-inversions of the space group. Such sets of orientations are called *orientation orbits*. The indices of the orientations in each orientation orbit are listed together and the indices of orientations in different orientation orbits are separated by a single or double horizontal line.

*Example*s

(1) In all cases, except for orthorhombic cases, different orientation orbits are separated by a double horizontal line, e.g. or the space group P23 (No. 195) one finds in the first column:

(001)
(100)
(010)
44444
(111)
(&&1)
(1&&)
(&1&)

Each orientation orbit contains those orientations which are related by the rotations of the space group P23.

(2) For orthorhombic space groups there are three orientation orbits each consisting of a single orientation (001), (100), and (010). These are, in general separated by a single horizontal line, e.g. For the space group P222 (No. 16) one finds in the first column:

(001)
)))))
(100)
)))))
(010)

In this case there is one orientation in each orientation orbit as the rotations of the space group P222 leave each orientation invariant.

(3) For orthorhombic space groups with the point group mm2, the orientation orbit (001) is separated by a double horizontal line, e.g. for the space group Fdd2 (No. 43) one finds in the first column:

(001)
44444
(100)
)))))
(010)

3. Conventional basis of the scanning group

For a given space group and orientation of a transecting plane, the *scanning group* is that equi-translational subgroup of the space group the elements of which leave invariant the orientation of the given plane. The scanning group is central in the methodology used to derive the sectional layer groups. In the second column, for each planar orientation given in the first column, the conventional basis vectors **a'**, **b'**, and **d**, of the scanning group are given in terms of the conventional basis vectors **a**, **b**, and **c** , of the space group. The basis vectors **a'** and **b'** define the translation subgroup of the sectional layer groups of planes of this orientation. The vector **d** defines the *scanning direction* and is used to define the position of the plane within the crystal .

For rhombohedral space groups the scanning group's conventional basis vectors are given in terms of the space group's hexagonal coordinate system. For some orientations of planes the various basis vectors are given in terms of the rhombohedral basis vectors $\mathbf{a}_r$, $\mathbf{b}_r$, and $\mathbf{c}_r$. These basis vectors in terms of the hexagonal basis vectors, and are given by:

$$\mathbf{a}_r = (2\mathbf{a}+\mathbf{b}+\mathbf{c})/3$$

$$\mathbf{b}_r = (-\mathbf{a}+\mathbf{b}+\mathbf{c})/3$$

$$\mathbf{c}_r = (-\mathbf{a}-2\mathbf{b}+\mathbf{c})/3$$

For cubic space groups the following symbolic abbreviations are used in defining the basis vector **d**:

$$\mathrm{J} = (\mathbf{a}+\mathbf{b}+\mathbf{c})$$

$$\mathrm{J}_1 = (\mathbf{a}-\mathbf{b}-\mathbf{c})$$

$$\mathrm{J}_2 = (-\mathbf{a}+\mathbf{b}-\mathbf{c})$$

$$\mathrm{J}_3 = (-\mathbf{a}-\mathbf{b}+\mathbf{c})$$

Examples

(1) For the planar orientation (100) and the space group P422 (No. 89):

a' **b'** **d**
))))))))))))))))))))))
b **c** **a**

(2) For the planar orientation (1&00) and the space group R32 (No. 155):

a' **b'** **d**
))))))))))))))))))))))
c -(**a**+**b**) -$\mathbf{b}_r$

(3) For the planar orientation (&1&) and the space group P23 (No. 195):

a' **b'** **d**
))))))))))))))))))))))
-a-b **b**+**c** J_2

4. The scanning group

The scanning group is given in the third column. The symbol for the scanning group is the same for all planar orientations in an orientation orbit provided one uses the corresponding conventional basis of each orientation. Consequently a single scanning group symbol is given for each orientation orbit. The space group symbol of the scanning group is given in the short Hermann-

Mauguin notation.

Example

For the space group $P4_2/mmc$ (No. 131) for the orientation orbit containing the orientations (100) and (010), one finds:

(100)	**b**	**c**	**a**	Pmmm
(010)	**-a**	**c**	**b**	

The scanning group symbol is Pmmm (No. 47) for both the planar orientation (100), in the conventional coordinate system **a'**,**b'**,**d** = **b**,**c**,**a** , and for the planar orientation (010), in the conventional coordinate system **a'**,**b'**,**d** = **-a**,**c**,**b**.

The space group symbol may not be the symbol for that space group in its standard setting. This is due to having the **a'** and **b'** basis vectors of the scanning group always representing the basis vectors of the resulting sectional layer group. The alternate setting symbols used are those listed in Table 4.3.1 of Section 4 of *ITC(1983)*.

Example

For the space group P&b2 (No. 117) and the orientation orbit containing the orientations (100) and (010), the scanning group is Pc2a. This is the space group Pba2 (No. 32) in the (a&b) setting.

Additional notations may be given to specify the origin of the scanning group.

Examples

1) For the space group $P4_232$ (No. 208) for the orientation orbit containing the orientation (001) the scanning group is listed as:

$P4_222$
(**a**'/2 or **b**'/2)

The origin of the scanning group is not at the origin **P** of the space group but at either **P** + **a**'/2 or **P** + **b**'/2.

2) For the space group I4/a&2/d (No. 230) for the orientation orbit containing the orientation (001) the scanning group is listed as

$I4_1/acd$
(origin 2)

The origin of the scanning group is the second origin choice as given in *ITC(1983)*.

In a few exceptional cases, the scanning group of the planes of an orientation orbit has a different origin for each of the planar orientations.

Example

For the space group $P2_13$ (No. 198) one finds:

(1 1 1)	with respect to the origin at **P**
(& & 1)	with respect to the origin at **P**+(**a**+**c**)/2
R3	
(1 & &)	with respect to the origin at **P**+(**b**+**a**)/2
(& 1 &)	with respect to the origin at **P**+(**c**+**b**)/2

For orthorhombic space groups and orientation orbits (001), (100), and (010), the scanning group is identical with the space group. However the symbol for the scanning groups may be different as they refer to different bases **a'**,**b'**,**d**. If the scanning group symbols are identical, only one scanning group symbol is given.

Examples

(1) For the space group C222 (No. 21) one finds:

(001)	**a**	**b**	**c**	C222
(100)	**b**	**c**	**a**	B222
(010)	**c**	**a**	**b**	A222

Here, the scanning group symbol is distinct for each of the three orientation orbits.

(2) For the space group F222 (No. 22) one finds:

(001)	**a**	**b**	**c**	F222
(100)	**b**	**c**	**a**	
(010)	**c**	**a**	**b**	

Here, the scanning group symbol is identical for all three of the orientation orbits.

5. Linear orbit

To specify a plane transecting a crystal, one must give both its orientation and its position within the crystal. The orientation of the plane is specified in the first column of the tables . The position of a plane is specified in the column under *Linear Orbit s**d***. The position is specified by the point **P** + s**d** where the plane intersects the scanning direction, the direction of the vector **d**, that is, the position of the plane is specified by the value "s" of the vector s**d** which defines the point **P** + s**d**.

The infinite set of all parallel planes of a specific orientation which transect a crystal can be subdivided into subsets, called *linear orbits*. All parallel planes obtained by applying all elements of the space group to any one plane of a specified orientation constitute a single linear orbit. The positions of all planes in a linear orbit are specified by the set of vectors [s_1**d** + n**d**, s_2**d** + n**d**, ... , s_q**d** + n**d**] where "q" is a finite number, 0< s_i <1, i=1,2,...,q , and because of the periodicity of the crystal in the scanning direction, n 0 Z , the set of all integers.

In the *Linear Orbit s**d*** column the positions of the planes in each translation orbit is given, for typographical simplicity, by the position vectors [s_1**d**, s_2**d**, ... , s_q**d**] and if this set of vectors contains a single vector [s**d**] by s**d**, i.e. without the brackets. Using this notation with the vector **d**, the translation orbits are the same for all planar orientations listed in the first column of the same linear orbit entry row. For each orientation the corresponding vector **d** found in the second column is used.

Example

(1) For the space group I222 (No. 23), the linear orbit [0**d**, ½**d**] denotes the linear orbit [0**c**, ½**c**] for the orientation orbit (001), [0**a**, ½**a**] for the orientation orbit (100), and [0**b**, ½**b**] for the orientation orbit (010).

The sectional layer group of each plane is given in the fifth column on the same line as position vector s**d**. The sectional layer group of planes with a specific orientation and positions belonging to planes in the same linear orbit are of the same type but may be orientated in a different manner with respect to the scanning group's conventional coordinate system. Consequently, the position vectors of planes in the same linear orbit may be written on consecutive lines.

Examples

1) For the space group P4/m (No. 83) and planar orientation (001) one finds in the fourth column: 0**d**, ½**d**

[s**d**, -s**d**]

Here there are two linear orbits with fixed values of the parameter s, delineated by a comma, and an infinity of linear orbits denoted by [s**d**, -s**d**] with the variable parameter "s" taking on all values 0#s<1 except for the previously given fixed values. The first two linear orbits are written on the same line as the planes at these positions have the same sectional layer group.

2) For the space group $P4_2/m$ (No. 84) and planar orientation (001) one finds:

[0**d**, ½**d**]

[s**d**, -s**d**]

Here the planes at positions 0**d** and ½**d** belong to the same linear orbit.

3) For the space group $P4_{1}22$ (No. 91) and planar orientation (001) one finds:

[0**d**, 1/2**d**;
1/4**d**, 3/4**d**]

[1/8**d**, 5/8**d**;
3/8**d**, 7/8**d**]

[±s**d**, (±s+1/4)**d**, (±s+1/2)**d**, (±s+3/4)**d**]

Here there are two linear orbits with fixed parameters. Each orbit is listed over two lines, delineated by a semi-colon, as the sectional layer groups of the planes at the positions in each orbit are of the same type but orientated in a different manner with respect to the scanning groups conventional coordinate system.

4) In a few cases, for typographical simplicity of the tables, the listings under the fourth column are subdivided into two columns each containing a single linear orbit. For the space group R& (No. 148) and orientation (0001) one finds:

[0**d**, [1/2**d**,
1/3**d**, 5 5/6**d**,
2/3**d**] 1/6**d**]

Planes at positions listed on each line have the same sectional layer group.

6. Sectional layer group

The sectional layer group of each plane is given in the fifth column on the same line as the position of the plane given in the fourth column. The symbol for the sectional layer group is made up of three parts:

1) A short Hermann-Mauguin symbol of a layer group. This may be the symbol in the standard setting or in an alternate setting as given in Section 5.

2) The Hermann-Mauguin symbol is followed, in brackets, by a vector **v** used to determine the origin of the sectional layer group within the space group's coordinate system. The origin of the sectional layer group is at **P** + s**d** + **v** , where **P** is the origin of the space group and s**d** is the position of the plane along the scanning direction given in the fourth column, and **v** is a vector in the plane. If **v** = **0** then no vector is given.

3) A symbol Ln where n is the serial number of the layer group type as listed in Section 5.

Examples

1) For the space group P&m1 (No. 164) one finds

listed:

(*hkil*)	**a' b' d**	s**d**	Sectional Layer Group	
(0001)	**a b c**	0**d**,1/2**d**	p&m1	L72
		[s**d**,-s**d**]	p3m1	L69

In this case the vector **v**=**0** and no additional vector is given between the Hermann-Mauguin symbols and the serial number of the layer group type.

2) For the space group R&m (No. 166) one finds listed:

(*hkil*)	**a' b' d**	s**d**		Sectional Layer Group	
(0001)	**a b c**	[0**d**,	[1/2**d**	p&m1	L72
		1/3**d**, 5	5/6**d**	p&m1 [(2**a'**+**b'**)/3]	L72
		2/3**d**]	1/6**d**]	p&m1 [(**a'**+2**b'**)/3]	L72

Here the origin of the sectional layer group varies according to the position of the plane.

3) For the space group $P222_1$ (No. 17) one finds listed:

(*hkl*)	**a' b' d**	s**d**	Sectional Layer Group	
(100)	**b c a**	0**d**,1/2**d**	$p22_12$	L20

The sectional layer group $p22_12$ is a layer group of the type L20 but not in the standard orientation. It is the layer group $p2_122$ in the (*b,&,c*) setting, see Table 2.6.1 and the diagrams for the layer group type L20 in Section 5.

Two additional symbols are used in the short Hermann-Mauguin symbol of the sectional layer group. If the conventional cell basis vectors **a'** and **b'** of the scanning group are those of a centered lattice and the conventional cell of the sectional layer group is primitive, then the sectional layer group's lattice symbol is denoted by p̂. The basis vectors of this primitive lattice are the vectors (**a'**+**b'**)/2 and (**a'**-**b'**)/2. If the conventional cell basis vectors **a'** and **b'** of the scanning group are those of a primitive lattice and the conventional cell of the sectional layer group is centered, then the sectional layer group's lattice symbol is denoted by ĉ. The basis vectors of this centered lattice are (**a'**+**b'**) and (**a'**-**b'**).

Examples

1) For the space group C222 (No. 21) one finds listed:

(*hkl*)	**a' b' d**	Scanning group	s**d**	Sectional Layer Group	
(001)	**a b c**	C222	0**d**,1/2**d**	c222	L22
			[s**d**,-s**d**]	p̂112	L3

The scanning group has a centered conventional unit cell while the sectional layer group of the planes at s**d** and -s**d** have a primitive conventional unit cell.

2) For the space group $P4_222$ (No. 93) one finds listed:

(*hkl*)	**a' b' d**	Scanning group	s**d**	Sectional Layer Group	
(001)	**a b c**	$P42_22$	[0**d**,1/2**d**]	p222	L19
			[1/4**d**,3/4**d**]	ĉ222	L22
			[±s**d**,(±s**d**+1/2)**d**]	p112	L3

The scanning group has a primitive conventional unit cell while the sectional layer group of the planes at 1/4**d** and 3/4**d** have a centered conventional unit cell.

Explicit Tables - variable indices

Explicit tables are also given for triclinic and monoclinic space groups and planes with orientations having variable indices. For the triclinic space groups each orientation orbit consists of a single orientation whose indices are given by *(h,k,l)*. The indices *h*, *k*, and *l* denote variable indices, i.e. any trio of integer values defines a single orientation. For any choice of indices, listed under the conventional basis of the scanning group is "any admissible choice." That is, one can choose any set of vectors **a**' and **b**' which are basis vectors for the translation group of the plane of orientation *(h,k,l)* and any vector **d** such that the vectors **a**',**b**',**d** are a set of basis vectors for the translation group of the triclinic space group. The form and content of the remainder of the triclinic explicit tables are the same as discussed above.

For monoclinic space groups, in addition to the orientation (001), with fixed indices, only orientations (mn0) are considered. Planes of any other orientation have the symmetry of the trivial layer group *p*1 (L1). Each orientation orbit consists of a single orientation (mn0) where m and n take on specific integer values. For each orientation (mn0) the conventional basis of the scanning group is:

a'	**b'**	**d**
c	n**a**-m**b**	p**a**+q**b**

For the vectors **a**',**b**',**d** to be a conventional basis of the scanning group, the condition nq+mp=1 on the integers n,m,p,q must be satisfied. This implies a condition on the parity of the four integers. There are only six combinations of parity which satisfy this condition:

	n	q	m	p
1)	odd	odd	even	even
2)	even	even	odd	odd
3)	even	odd	odd	odd
4)	odd	odd	odd	even
5)	odd	even	odd	odd
6)	odd	odd	even	odd

The symbol of the corresponding scanning group may depend on the parity of these four integers.

Examples

1) For the space group Cc (No. 9), cell choice 1, A11a, the symbol of the corresponding scanning group is different for each of the six combinations of parity:

	a'	**b'**	**d**
(mn0)	**c**	n**a**-m**b**	p**a**+q**b**

n odd m even p even q odd	Bb11
n even m odd p odd q even	Cc11
n even m odd p odd q odd	Cn11
n odd m odd p even q odd	Bn11
n odd m odd p odd q even	Ic11
n odd m even p odd q odd	Ib11

2) For the space group Pm (No. 6), the symbol for the scanning group is the same for all six combinations of parity:

	a'	**b'**	**d**	
))				
(mn0)	**c**	n**a**-m**b**	p**a**+q**b**	Pm11

3) For the space group Cm (No. 8), cell choice 1, A11m, the symbol for the scanning group is the same for pairs of the six combinations of parity:

	a'	**b'**	**d**
))			
(mn0)	**c**	n**a**-m**b**	p**a**+q**b**

n odd p even q odd	Bm11
n even m odd p odd	Cm11
n odd p odd	Im11

An abbreviated notation is used to denote the parity of two quadruples of four integers: In the upper case, the integer m is implied and can be either even or odd. In the middle case, q is implied and again can be either even or odd. In the bottom case, the pair of integers m and q are respectfully either odd and even, or even and odd.

The remainder of these monoclinic tables are in the format and content as discussed above.

Auxiliary Tables

For higher than triclinic and monoclinic space groups and orientations with variable indices, scanning tables are constructed from auxiliary tables. There are two types of auxiliary tables, the *arithmetic class* and *centering type* tables. These tables provide the information to construct the scanning tables from the explicit scanning tables of the (mn0) orientation orbits of monoclinic space groups.

The arithmetic class tables give for each space group and each orientation orbit the monoclinic space group from which the scanning tables are constructed for that orientation orbit. The layer group for planes of orientations not listed is the trivial layer group *p*1 L01.

Example

For the orthorhombic arithmetic class 222C one finds the table:

Serial No.	20	21
Group	$C222_1$	C222
(hk0) (&k0)	$P112_1$	P112
(0mn) (0&n)	B112	B112
(n0m) (n0&)	A112 (**c**/4)	A112

A vector in parenthesis under a monoclinic space group symbol (e.g. A112) indicates that when one constructs the scanning tables of the space group ($C222_1$) for the orientations in the first column ((nom) and (n0&)) one uses the information in the monoclinic (A112) space group scanning tables but relative not to the origin **P** of the space group ($C222_1$) but to **P** + **v**, where **v** (= **c**/4) is the vector under the monoclinic space group symbol (A112).

For the space group C222 (no. 21) one constructs the scanning table for the first orientation orbit containing the orientations (hk0) and (&k0) from the scanning table for the (mn0) orbit of the monoclinic space group P112. The scanning tables for the second and third orientation orbits are constructed from scanning tables of the monoclinic groups B112 and A112, respectively.

The actual construction of the scanning tables is in two steps: The first step: From the scanning table of the orientation orbit (mn0) of the monoclinic space group one replaces the orientation orbit with that orientation orbit listed in the arithmetic class table.

Example

For the space group C222 (No. 21) and orientation orbit consisting of the orientations (hk0) and (&k0) one uses the scanning table for the (mn0) orbit of the monoclinic space group P112. One replaces in the latter (mn0) with the orientations (hk0) and (&k0).

The second step is to replace the conventional basis of the (mn0) orbit in the monoclinic space group scanning table with the conventional bases for the orientations of the new orientation orbit. The remainder of the table stays the same.

The conventional basis of the (mn0) orbit of the monoclinic scanning group is replaced by new conventional bases given in the centering tables. If the (mn0) orbit in the monoclinic scanning tables has been replaced by orientations given in the first column of the centering tables, then the conventional basis of the scanning group for the orientation (mn0) found in the monoclinic scanning tables is replaced by the conventional bases found in the second through fourth columns of the centering table.

Example

For centered orthorhombic space groups one has the Centering Type C table:

Orientation orbit	Conventional basis of the scanning group			Auxiliary basis of the scanning group		
(hkl)	**a'**	**b'**	**d**	$\hat{\mathbf{a}}$	$\hat{\mathbf{b}}$	$\hat{\mathbf{c}}$
(hk0)	**c**	$n\hat{\mathbf{a}}-m\hat{\mathbf{b}}$	$p\hat{\mathbf{a}}+q\hat{\mathbf{b}}$	(**a**-**b**)/2	(**a**+**b**)/2	**c**
$(\bar{h}k0)$	**c**	$n\hat{\mathbf{a}}+m\hat{\mathbf{b}}$	$-p\hat{\mathbf{a}}+q\hat{\mathbf{b}}$			

h even, k odd or h odd, k even: n=h+k, m=h-k
h,k odd: n=(h+k)/2, m=(h-k)/2

(0mn)	**a**	n**b**-m**c**	p**b**+q**c**	**b**	**c**	**a**
$(0\bar{m}n)$	**a**	n**b**+m**c**	-p**b**+q**c**			
(n0m)	**b**	n**c**-m**a**	p**c**+q**a**	**c**	**a**	**b**
$(n0\bar{m})$	**b**	n**c**-m**a**	-p**c**+q**a**			

The replacement conventional basis of the scanning group for each orientation listed in the first column is given in the second through fourth column. In some cases, as for the first two orientations listed, the conventional basis is given in terms of an auxiliary basis listed in the last three columns, and the relationships between the integers used in listing the conventional basis and the integers defining the orientation are additionally given.

As an example we shall construct the scanning tables for orientations with variable indices for the space group C222 (No. 21). From the arithmetic class 222C table (shown above) we have three orientation orbits each with a different corresponding monoclinic group. The first orientation orbit, with orientations (hk0) and (&k0) one uses the scanning table for the (mn0) orientation of the monoclinic space group P112. This scanning table is:

(mn0)	**c**	n**a**-m**b**	p**a**+q**b**	P211	0**d**,½**d**	*p*211	L08
					[s**d**,-s**d**]	*p*1	L01

The oriention (mn0) is replaced with the two orientations (hk0) and (&k0)and the conventional basis of the scanning group is replaced with the conventional bases corresponding to the two orientations (hk0) and (&k0) found in the orthorhombic C centering table (shown above). Consequently the scanning table for the space group C222 (No. 21) and orientation orbit containing the two orientations (hk0) and (&k0) is:

(hk0)	**c**	n**$a**-m**$b**	p**$a**+q**$b**	P211	0**d**,½**d**	*p*211	L08
(&k0)	**c**	n**$a**+m**$b**	-p**$a**+q**$b**		[s**d**,-s**d**]	*p*1	L01

where: **$a**=(**a**-**b**)/2 and **$b**=(**a**+**b**)/2
n=h+k and m=h-k if h even and k odd or h odd and k even
n=(h+k)/2 and m=(h-k)/2 if h and k both odd

For the second orientation orbit containing the orientation (0mn) and $(0\bar{m}n)$ one uses the scanning table for the (mn0) orientation of the monoclinic space group B112, i.e. the space group A112 (No. 5) in cell choice 2. This scanning table is:

(mn0)	**c**	n**a**-m**b**	p**a**+q**b**						
				n odd	m even	C211	0**d**,½**d**	c211	L10
					q odd		[s**d**,-s**d**]	$\bar{p}1$	L01
					m odd	I211	[0**d**,½**d**]	p211	L08
					q odd		[1/4**d**,3/4**d**]	$p2_111$(b'/4)	L09
							[±s**d**,(±s+½)**d**]	p1	L01
					m odd	B211	[0**d**,½**d**]	p211	L08
				p odd	q even		[1/4**d**,3/4**d**]	$p2_111$	L09
							[±s**d**,(±s+½)**d**]	p1	L01

The orientation (mn0) is replaced with the two orientations (0mn) and $(0\bar{m}n)$and the conventional basis of the scanning group is replaced with the conventional bases corresponding to the two orientations (0mn) and $(0\bar{m}n)$ found in the orthorhombic C centering table. Consequently the scanning table for the space group C222 (No. 21) and orientation orbit containing the two orientations (0mn) and $(0\bar{m}n)$ is:

(0mn)	**a**	n**b**-m**c**	p**b**+q**c**						
$(0\bar{m}n)$	**a**	n**b**+m**c**	-p**b**+q**c**						
				n odd	m even	C211	0**d**,½**d**	c211	L10
					q odd		[s**d**,-s**d**]	$\bar{p}1$	L01
					m odd	I211	[0**d**,½**d**]	p211	L08
					q odd		[1/4**d**,3/4**d**]	$p2_111$(b'/4)	L09
							[±s**d**,(±s+½)**d**]	p1	L01
					m odd	B211	[0**d**,½**d**]	p211	L08
				p odd	q even		[1/4**d**,3/4**d**]	$p2_111$	L09
							[±s**d**,(±s+½)**d**]	p1	L01

For the third orientation orbit containing the orientation (n0m) and (n0m̄) one uses the scanning table for the (mn0) orientation of the monoclinic space group A112, i.e. the space group A112 (No. 5) in cell choice 1. This scanning table is:

(mn0)	**c**	n**a**-m**b**	p**a**+q**b**					
				n odd	B211	[0**d**,½**d**]	p211	L08
				p even q odd		[1/4**d**,3/4**d**]	p$2_1$11	L09
						[±s**d**,(±s+½)**d**]	p1	L01
				n even m odd	C211	0**d**,½**d**	c211	L10
				p odd		[s**d**,-s**d**]	p1	L01
				n odd	I211	[0**d**,½**d**]	p211	L08
				p odd		[1/4**d**,3/4**d**]	p$2_1$11(b'/4)	L09
						[±s**d**,(±s+½)**d**]	p1	L01

The orientation (mn0) is replaced with the two orientations (n0m) and (n0m̄)and the conventional basis of the scanning group is replaced with the conventional bases corresponding to the two orientations (n0m) and (n0m̄) found in the orthorhombic C centering table. Consequently the scanning table for the space group C222 (No. 21) and orientation orbit containing the two orientations (n0m) and (n0m̄) is:

(n0m)	**b**	n**c**-m**a**	p**c**+q**a**					
(n0m̄)	**b**	n**c**+m**a**	-p**c**+q**a**					
				n odd	B211	[0**d**,½**d**]	p211	L08
				p even q odd		[1/4**d**,3/4**d**]	p$2_1$11	L09
						[±s**d**,(±s+½)**d**]	p1	L01
				n even m odd	C211	0**d**,½**d**	c211	L10
				p odd		[s**d**,-s**d**]	p1	L01
				n odd	I211	[0**d**,½**d**]	p211	L08
				p odd		[1/4**d**,3/4**d**]	p$2_1$11(b'/4)	L09
						[±s**d**,(±s+½)**d**]	p1	L01

Consequently, for the space group C222 and orientation orbits with variable values, the scanning tables are:

(hk0)	**c**	n$\hat{\mathbf{a}}$-m$\hat{\mathbf{b}}$	p$\hat{\mathbf{a}}$+q$\hat{\mathbf{b}}$		P211	0**d**,½**d**	*p*211	L08
($\bar{h}$k0)	**c**	n$\hat{\mathbf{a}}$+m$\hat{\mathbf{b}}$	-p$\hat{\mathbf{a}}$+q$\hat{\mathbf{b}}$			[s**d**,-s**d**]	*p*1	L01

where: $\hat{\mathbf{a}}$=(**a**-**b**)/2 and $\hat{\mathbf{b}}$=(**a**+**b**)/2
n=h+k and m=h-k if h even and k odd or h odd and k even
n=(h+k)/2 and m=(h-k)/2 if h and k both odd

(0mn)	**a**	n**b**-m**c**	p**b**+q**c**						
(0$\bar{m}$n)	**a**	n**b**+m**c**	-p**b**+q**c**						
				n odd	m even	C211	0**d**,½**d**	c211	L10
					q odd		[s**d**,-s**d**]	$\hat{p}$1	L01
					m odd	I211	[0**d**,½**d**]	p211	L08
					q odd		[1/4**d**,3/4**d**]	p$2_1$11(b'/4)	L09
							[±s**d**,(±s+½)**d**]	p1	L01
					m odd	B211	[0**d**,½**d**]	p211	L08
				p odd	q even		[1/4**d**,3/4**d**]	p$2_1$11	L09
							[±s**d**,(±s+½)**d**]	p1	L01
(n0m)	**b**	n**c**-m**a**	p**c**+q**a**						
(n0$\bar{m}$)	**b**	n**c**+m**a**	-p**c**+q**a**						
					n odd	B211	[0**d**,½**d**]	p211	L08
				p even	q odd		[1/4**d**,3/4**d**]	p$2_1$11	L09
							[±s**d**,(±s+½)**d**]	p1	L01
				n even	m odd	C211	0**d**,½**d**	c211	L10
					p odd		[s**d**,-s**d**]	$\hat{p}$1	L01
					n odd	I211	[0**d**,½**d**]	p211	L08
					p odd		[1/4**d**,3/4**d**]	p$2_1$11(b'/4)	L09
							[±s**d**,(±s+½)**d**]	p1	L01

For the auxiliary tables in the case of the space group C222$_1$, for the orientations (hk0), ($\bar{h}$k0), (0mn) and (0$\bar{m}$n), the tables are constructed as above. For the (n0m) and (n0$\bar{m}$) orientations, in the orthorhombic arithmetic class 222C table one finds the monoclinic space group A112 with the vector **v** = **c**/4 below. Once constructs the scanning tables for these orientations from the scanning tables of the monoclinic space group in the same manner as in the examples above. This gives the auxiliary scanning tables of the space group $C222_1$:

a'	b'	d	Scanning group	s**d**	Sectional layer group	
(n0m)	**b**	n**c**-m**a** p**c** - q**a**				
$(n0\bar{m})$	**b**	n**c**+m**a** -p**c** +q**a**				
(**c**/4)		n odd	B211	[0**d**,½**d**]	p211	L08
		p even q odd		[1/4**d**,3/4**d**]	$p2_111$	L09
				[±s**d**,(±s+½)**d**]	p1	L01
		n even m odd	C211	0**d**,½**d**	c211	L10
		p odd		[s**d**,-s**d**]	p1	L01
		n odd	I211	[0**d**,½**d**]	p211	L08
		p odd		[1/4**d**,3/4**d**]	$p2_111$(b'/4)	L09
				[±s**d**,(±s+½)**d**]	p1	L01

We have added, under the orientations, the vector **v**. This means that the information on the right, i.e. the scanning groups, linear orbits s**d** and the sectional layer groups are with respect to, not the origin **P** of the space group $C222_1$, but to a new origin taken at **P** + **v**.

In Figure 1 we give the symmetry diagram of the space group $C222_1$ and in Figure 2, The same diagram with only the two-fold rotation axes which are symmetries of planes of the orientations (n0m) and $(n0\bar{m})$. We also show the origin **P** of the space group $C222_1$ and the point **P** + **v**, where **v** = **c**/4 . Note that with respect to a new origin taken at P + c/4 a two-fold rotation axis passes through the new origin.

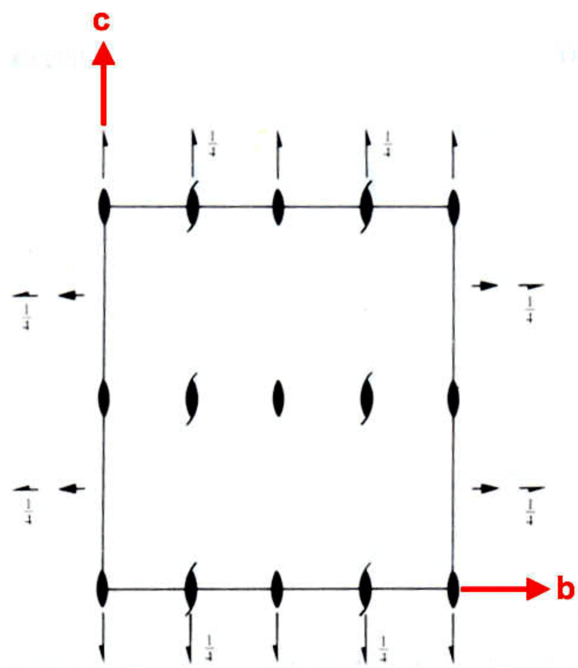

Figure 1

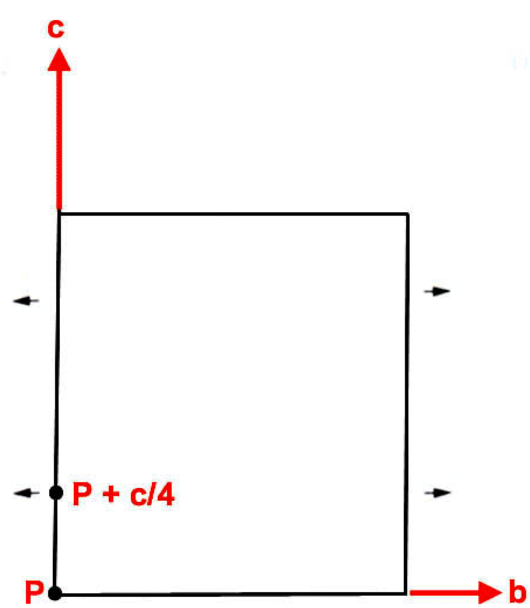

Figure 2