One-piece Faraday generator: A paradoxical experiment from 1851

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In the conventional Faraday generator a conducting disk rotates in an axial magnetic field. If the disk is replaced by a cylindrical permanent magnet that supplies its own magnetic field, the effect is identical. It follows that any moving magnet generates an induced electromotive force due to the presence of its own field: this generalization leads to an apparent paradox in the case of translational motion for it implies the possibility that an observer in an inertial frame could measure his absolute velocity.

The Faraday generator[1] (Fig. 1) comprises a circular conducting disk rotating about its axis in the presence of an axial magnetic field. Although there is no doubt about the induction effect and its magnitude, the theory of the device has been the subject of considerable debate between the advocates of "flux-cutting" (or motional emf) and supporters of "flux-linking".[2] The complete explanation of induction phenomena in general, and the Faraday generator in particular, was given relatively recently by Bewley[3] and Corson.[4]

Perhaps the most straightforward method for dealing with electromagnetic induction was presented by Feynman,[5] namely, by stating that the correct physics is always given by the Lorentz force,

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad (1)$$

and the Maxwell equation,

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t. \qquad (2)$$

All quantities are to be measured in the same reference frame of course.

Now suppose that a conducting loop is moving in the reference frame. Let element $d\mathbf{l}$ move with velocity $\mathbf{v}$, and let S be an area bounded by the loop. Then the electromotive force is the work done in taking unit charge around the loop (instantaneously),

$$\begin{aligned}\mathcal{E} &= \frac{1}{q} \oint \mathbf{F} \cdot d\mathbf{l} \\ &= \oint \mathbf{E} \cdot d\mathbf{l} + \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}\end{aligned}$$

from Eq. (1).
Using Stokes' theorem and Eq. (2) we obtain

$$\begin{aligned}\mathcal{E} &= \int (\nabla \times \mathbf{E}) \cdot d\mathbf{S} + \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} \\ &= -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}. \qquad (3)\end{aligned}$$

Equation (3) gives the emf as the sum of a "transformer emf" obtained by integration of $-\partial \mathbf{B}/\partial t$ over the instantaneous area S, and a "motional emf" from $\mathbf{v} \times \mathbf{B}$.

Applying the divergence theorem to the volume swept out by S in time dt, it follows from Eq. (3) that

$$\mathcal{E} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{S} \qquad (4)$$

which is Faraday's Law ($\mathcal{E} = -\dot{\phi}$ where ϕ is the flux through S). In the usual textbook (and historical) approach, Eq. (4) is taken as fundamental and Eq. (2) is derived from it. An equivalent derivation to ours is given by Landau and Lifshitz[6] where the calculation is performed in the frame of reference of the conductor.

While Eq. (4) is commonly used in the interpretation of electromagnetic phenomena, there are situations where the evaluation of the total derivative is not obvious. This has led to incredible confusion[2] and we would advocate the following rule—"when in doubt, use Eq. (3)," and we shall use this form of Faraday's Law throughout. Those who are strongly addicted to the use of Eq. (4) are invited to try it out on some of the famous paradoxes of electromagnetism.[7,8]

When using Eq. (3) different observers attached to different reference frames may disagree over the details, but agree on the answer: for example, for a loop moving in an inhomogeneous field the emf is generated by the "transformer" term for an observer on the loop, but comes from the motional emf for an observer in the laboratory frame. Applying Eq. (3) to the Faraday generator (in a uniform magnetic field) as viewed from the laboratory frame, then $\partial \mathbf{B}/\partial t = 0$ everywhere, and the emf arises from the second term because of the rotation of the disk. A simple integration along the radius of the disk yields,

$$\mathcal{E} = (1/2)\, R^2 B \omega, \qquad (5)$$

where R is the disk radius and ω the angular frequency. For an observer on the rotating disk, $\partial \mathbf{B}/\partial t = 0$ as before, but now the rest of the circuit is rotating in the field, and Eq. (5) again follows from integration of the second term of Eq. (3), though the new observer will have to use his own values for the parameters.

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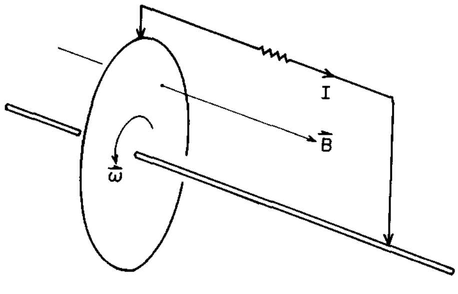

Fig. 1. The Faraday generator. The axial magnetic field is uniform and constant.

ONE-PIECE FARADAY GENERATOR

Suppose that, following Faraday,[9] we replace the disk of Fig. 1 by a conducting magnetized disk that supplies its own **B** parallel to the axis, so that the externally applied field is no longer required. The device is now a one-piece Faraday generator (or motor, if driven from a current source). Almost any physicist or electrical engineer when faced with this device will immediately claim that it will not work. It is too much like trying to lift yourself off the ground by pulling on your own bootlaces! Yet it has been known since 1851 that the device does indeed generate an emf.

To confound the skeptics we constructed a one-piece generator which produced the results shown in Fig. 2. The generator comprised an Alnico cylindrical bar magnet, about 1-cm diam. and 4-cm long. The magnet was mounted in an insulating collar of Bakelite, and rotated at various rates in a lathe. Contacts to the magnet was made by means of pointed carbon brushes, one held in the tailstock of the lathe and making axial contact, and the other brush held against the perimeter of the magnet. Both brushes were insulated from the body of the lathe. Rotational speed was measured by a strobe light and the induced emf was measured by a digital voltmeter measuring to $\pm 10\ \mu$V.

The systematic difference between readings for forward and reverse rotation seen in Fig. 2 is probably the result of thermoelectric emf's at the sliding contacts. The observed difference is consistent with the fact that reversing the direction of rotation reverses the induced emf, but would leave the thermoelectric effect as before. Also the difference increases with increasing rotation rate, which would be expected from the increased generation of heat at the contacts.

The results shown in Fig. 2 were taken with the perimeter brush in the central region of the rotating magnet. For measurements made at the end of the magnet the observed emf's were a factor of roughly four or five times smaller (which would be expected because of the flux loss through the sides of the magnet as one proceeds from the center to the ends). The data were less reliable for the end position both because of the difficulty of maintaining smooth contact and because of the sensitivity of the reading to location (due to the rapid change of enclosed flux with axial distance in this vicinity).

Equation (2) predicts the emf with no adjustable parameters: for the present case it becomes

$$\mathcal{E} = 4.6 \times 10^{-6}\,\omega \qquad (\mathrm{V}). \qquad (6)$$

There is an uncertainty of about $\pm 10\%$ in the numerical constant that comes entirely from the estimate of B (= 0.40 T) which was calculated from an integration of the flux over one half of the cylinder with the aid of a Hall probe. A line

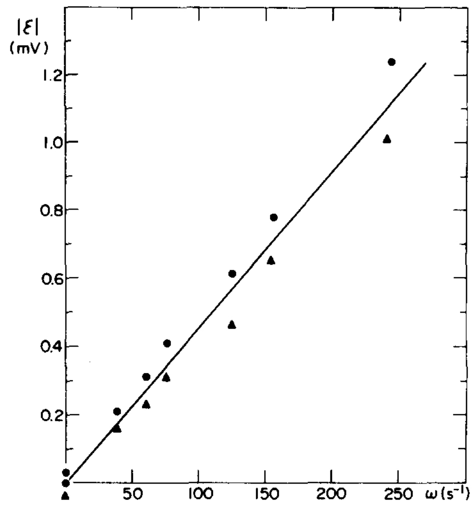

Fig. 2. The emf generated in a rotating magnet as a function of angular velocity. The symbols show counterclockwise (●) and clockwise (▲) rotation. The solid line is given by Eq. (6).

with slope corresponding to Eq. (6) and passing through the mean value of the observed emf at zero rotation speed is the theoretical line shown on Fig. 2: the agreement between this line and the data points is better than one has a right to expect.

The results shown in Fig. 2 confirm the predictions of Eq. (3) in a very satisfactory fashion: the calculation based on the $\mathbf{v} \times \mathbf{B}$ term is valid regardless of the source of the **B** field, and in particular when that source is the permanent magnetism of the rotating disk itself.

This result was well known to early writers on this subject,[10] but somehow it has been neglected in recent decades presumably because the Faraday generator in its conventional form created sufficient problems. The debate centered instead on the flux-cutting argument versus the flux-linking protagonists in connection with the conventional generator, so that Bewley[3] and Corson[4] concentrate on that problem too. Textbooks seem to mention only the conventional generator, although its simpler one-piece counterpart now seems more interesting.

It seems strange at first sight that the one-piece generator was not regarded as paradoxical in earlier times[10] whereas a physicist today is quite startled by it. Early workers were strongly addicted to the notion that lines of force are tangible things attached to the magnet: the one-piece generator was readily explained by the fact that these lines of force from the rotating magnet are cutting the external circuit and *not* the magnet. Our explanation, that the emf arises from $\mathbf{v} \times \mathbf{B}$ *within* the magnet is utter nonsense to anyone who believes in the objective reality of lines of force.[11]

On the other hand, if the one-piece generator is viewed from the rotating frame, the emf does indeed arise from external $\mathbf{v} \times \mathbf{B}$ forces—the two arguments coincide and lines of force are vindicated! Or are they? This raises an interesting point. The advocates of "flux-cutting" were satisfied if the explanation worked in any one frame, and failed to recognize that a universal law must work for all frames.

ANOTHER PARADOX

Why does the one-piece Faraday generator seem so paradoxical today, when the resolution of the paradox is so obvious in terms of Eq. (3) for observers in either frame of

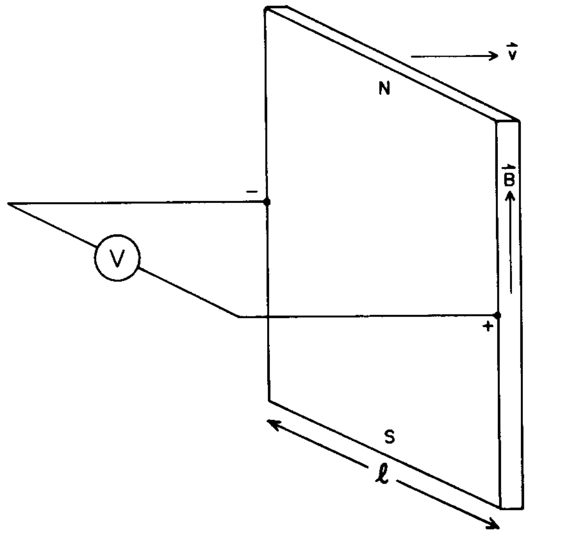

Fig. 3. Moving magnet. Does the voltmeter detect an emf resulting from the motion?

reference? The reason seems to be that the explanation of the one-piece generator implies that *any* magnet moving in its own field must generate an emf and in particular that includes linear translational motion of the magnet. Motion must be defined as being relative to something. This means that the observed emf would depend on the particular laboratory reference frame used. However, the emf can be measured by a voltmeter reading on which all observers must agree. If there is no error in this chain of reasoning then the observed emf can only be a measure of the absolute velocity of the magnet in space, violating the basic axiom of the Special Theory of Relativity.

To be more specific, consider the moving magnet shown in Fig. 3. The magnet is a conductor moving in the presence of its own magnetic field **B**. By making the magnet long enough, the returning magnetic field through space around the magnet can be made as small as desired, so that there are no compensating induced effects from the motion of the voltmeter and its connecting leads. Hence, in the laboratory frame there appears to be an emf in the circuit of magnitude lvB.

On the other hand, in the moving frame nothing is happening, there is no $\partial\mathbf{B}/\partial t$, no $\mathbf{v}\times\mathbf{B}$, and therefore no emf. The discrepancy arises because the transformer emf term of Eq. (3) is not zero in the laboratory frame. Consider a horizontal loop that is fixed in the laboratory frame, and passes through the magnet at a given instant (Fig. 4). At the edge of the magnet, **B** is changing, though the variation is a step-function in time. The integral $\int(\partial\mathbf{B}/\partial t)\cdot d\mathbf{S}$ for the plane of the loop therefore involves a delta function at the edge of the magnet, but when evaluated it gives a magnitude lvB. Application of Lenz's Law to the loop of Fig. 4 shows that the transformer emf must be counterclockwise while the motional emf is clockwise, and the resultant is therefore zero as observed in the moving frame.

The resolution of our paradox is briefly mentioned in a footnote of a paper by Pugh[12] where he is discussing another infamous paradox of electromagnetism that was originated by Hering.[13] A detailed treatment of the whole question would require the relativistic transformation of the fields and field equations (as discussed by Webster[14]). This author also warns that in calculating charge distributions, errors may arise when this is done from the point of view of an

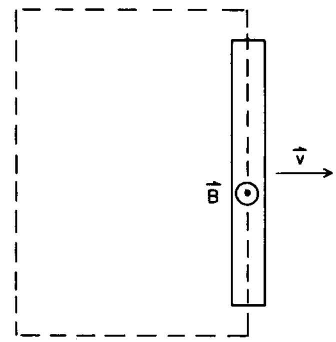

Fig. 4. Magnet of Fig. 3 viewed from above.

observer in a noninertial frame. However, he shows that this does not affect the calculation of emf's in closed loops, so with this limitation one is free to look at situations in electromagnetic induction from the viewpoint of a rotating observer.

Finally, returning to the one-piece Faraday generator, why is an emf obtained for the rotating system when there is no emf in the case of translational motion? The reason is that a $\mathbf{v}\times\mathbf{B}$ contribution is present in both situations, but in translational motion (Fig. 4) the moving boundary of the magnet results in a cancelling term from $\partial\mathbf{B}/\partial t$. For the rotating disk the boundary of the magnet does not change in either reference frame so there is no $\partial\mathbf{B}/\partial t$ term for either observer.

ACKNOWLEDGMENT

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[1]M. Faraday, Philos. Trans. R. Soc. 1832, p. 125. Also *Faraday's Diary*, edited by T. Martin (Bell, London, 1932), Vol. I, p. 384, paragraph 117. Note: the descriptions in the Diary are often clearer than in the subsequent publication.

[2]See, for example, G. I. Cohn, Electr. Eng. (Am. Inst. Electr. Eng.) **68**, 441 (1949), and subsequent correspondence in the same journal, pp. 645, 1018, 1113, 1138, and 1139.

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[4]D. R. Corson, Am. J. Phys. **24**, 126 (1956).

[5]R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics, Vol. II*, (Addison-Wesley, Reading, MA, 1964), Secs. 17-1, 2.

[6]L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, New York, 1960), Sec. 49.

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[8]A. Nussbaum, Phys. Ed. **7**, 231 (1972).

[9]M. Faraday, Philos. Trans. R. Soc. 1852, p. 25. Also *Faraday's Diary* (see Ref. 1), Vol. V, p. 403, paragraph 11 371.

[10]C. Hering, J. Franklin Inst. **192**, 599 (1921).

[11]Few physicists take lines of force seriously any more: anyone with lingering doubts should consult J. Slepian, Am. J. Phys. **19**, 87 (1951); and J. W. Then, Am. J. Phys. **30**, 411 (1962).

[12]E. M. Pugh, Am. J. Phys. **32**, 879 (1964).

[13]C. Hering, AIEE Trans. **27**, 1341 (1908).

[14]D. L. Webster, Am. J. Phys. **29**, 262 (1961).