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Distinction of magnetic non-ferroelastic domains

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It is shown that there always exists a coordinate system in which components of property tensors that distinguish between the domains of a magnetic non-ferroelastic domain pair differ solely in the two domains by a change in sign. The 309 classes of twin laws of magnetic non-ferroelastic domain pairs are listed and the twin laws, which are given in a coordinate system where the tensor distinction is provided by a change in sign of tensor components, are specified. If the twin law is not given in such a coordinate system, then a new coordinate system, related by a rotation, is specified.

1. Introduction

The low-symmetry phase of ferroic materials consists of homogeneous regions called domains. The interior bulk structures of the domains are called domain states and in the continuum description differ only in orientation and/or handedness. The properties of domain states are described by property tensors. When observed by one experimental set-up, or described in one coordinate system, different domain states exhibit some property tensor components, denoted morphic or spontaneous, which are different. These morphic components influence the average properties of polydomain samples and enable one to observe domains (see Wadhawan, 2000, or Janovec & Přívratská, 2003).

Two domain states that have different spontaneous strain are denoted as a ferroelastic domain pair. Such domain pairs exhibit in a polarizing microscope different colors and are easily distinguished (Salje, 1990). On the other hand, two domain states in a non-ferroelastic domain pair possess the same spontaneous strain and cannot be observed in this way. Morphic components of other property tensors have to be used to distinguish these states. In this paper, we turn our attention to a special property of morphic tensor components in non-ferroelastic domain pairs which simplifies the analysis of tensor distinction of non-ferroelastic domains.

Wadhawan (1997, 2000) has tabulated examples of non-ferroelastic domain pairs distinguished by physical property tensors, *e.g.* chromium oxide distinguished by components of the magnetoelectric tensor (Newnham & Cross, 1974*a*,*b*) and $Pb_5Ge_3O_{11}$ (Toledano & Toledano, 1976) distinguished by components of spontaneous polarization, compliance and optical gyration tensors. For pairs of ferroelectric non-ferroelastic domains, a listing of optical property tensors that can distinguish the domains has been given (Janovec *et al.*, 1992). This was extended to all non-magnetic non-ferroelastic domain pairs in Janovec *et al.* (1993). Tabulations of the matrix form of physical property tensors (Litvin & Litvin, 1990, 1991) were used in listing the matrix forms of property tensors that distinguish non-ferroelastic magnetoelectric domain pairs (Litvin *et al.*, 1994). From these latter tabulations, one can find not only which property tensors distinguish between the domains but also which components of the property tensors provide the distinction. From these cases, a general rule seemed to be implied: components of property tensors that distinguish non-ferroelastic domains differ only by a sign in the two domains.

In §2, we review the properties of non-ferroelastic twin laws, those twin laws that describe the symmetry of non-ferroelastic domain pairs. In §3, we show that the above implied rule is not in general valid as it is not valid in all coordinate systems. We then show that for non-ferroelastic domain pairs one can always choose a coordinate system in which it is valid.

2. Non-ferroelastic twin laws

Consider a phase transition between phases of symmetry $\mathbf{G}$ and $\mathbf{F}$. The crystal splits into $n = |\mathbf{G}|/|\mathbf{F}|$ single domain states denoted by $S_1, S_2, \ldots, S_n$ and the symmetry group of each single domain state is denoted, respectively, as $\mathbf{F}_1 = \mathbf{F}, \mathbf{F}_2, \ldots, \mathbf{F}_n$. Writing the coset decomposition of $\mathbf{G}$ with respect to $\mathbf{F}$ as $\mathbf{G} = \mathbf{F} + g_2\mathbf{F} + \ldots + g_n\mathbf{F}$, for $i = 1, 2, \ldots, n$, $S_i = g_iS_1$, *i.e.* the orientation of the ith domain S_i is related to the orientation of the domain S_1 by the element g_i of this coset decomposition. For $i = 1, 2, \ldots, n$, the symmetry group $\mathbf{F}_i = g_i\mathbf{F}g_i^{-1}$, *i.e.* the groups $\mathbf{F}$ and $\mathbf{F}_i$ are conjugate groups. A domain pair is denoted by (S_i, S_k).

The strain tensor is a property tensor of the type $[V^2]$, a second-rank symmetric tensor. The form of this tensor depends only on the crystal family of the point group. We denote the crystal family of a point group $\mathbf{F}$ by Fam($\mathbf{F}$) and the form of the strain tensor invariant under Fam($\mathbf{F}$) by $[V^2]_{\mathrm{Fam}[\mathbf{F}]}$. As a group, we define Fam($\mathbf{F}$) as the holohedral group of the crystal family to which $\mathbf{F}$ belongs.

We consider a non-ferroelastic domain pair (S_i, S_k). Let **F** be the point-group symmetry of the domain S_i whose strain tensor is then $[V^2]_{Fam[\mathbf{F}]}$. Let g_{ik} be an operation that transforms domain S_i into S_k, *i.e.* $g_{ik}S_i = S_k$. Since the two domains have the identical spontaneous strain, g_{ik} leaves the tensor $[V^2]_{Fam[\mathbf{F}]}$ invariant and consequently $\langle \mathbf{F}, g_{ik}\rangle \subseteq \mathrm{Fam}(\mathbf{F})$, *i.e.* the group generated by **F** and the element g_{ik}, is equal to or a subgroup of Fam(**F**). As any subgroup **F** of Fam(**F**) is a normal subgroup, $\langle \mathbf{F}, g_{ik}\rangle = \mathbf{F} + g_{ik}\mathbf{F}$. Consequently, for a non-ferroelastic domain pair (S_i, S_k), in addition to the fact that $\mathbf{F}_i = \mathbf{F}_k$ (= **F**) and $g_{ik}S_i = S_k$, one has that $g_{ik}S_k = S_i$, *i.e.* that the element g_{ik} transforms not only the domain S_i into the domain S_k but also the inverse transformation of the domain S_k into the domain S_i. This follows from the fact that, in the group $\mathbf{F} + g_{ik}\mathbf{F}$, $g_{ik}^2 \in \mathbf{F}$ and consequently $g_{ik}S_k = g_{ik}(g_{ik}S_i) = g_{ik}^2 S_i = fS_i = S_i$, where f is an element of the group **F**. For non-ferroelastic domain pairs, the element g_{ik} can be chosen such that $g_{ik}^2 = e$, the identity element of **F**.

The twin law $\mathbf{J}_{ik}$ of a domain pair (S_i, S_k) is defined (Janovec, 1981) as

$$\mathbf{J}_{ik} = (\mathbf{F}_i \cap \mathbf{F}_k) + g_{ik}^*(\mathbf{F}_i \cap \mathbf{F}_k), \tag{1}$$

where g_{ik}^* is an element that interexchanges the two domains, *i.e.* $g_{ik}^*S_i = S_k$ and $g_{ik}^*S_k = S_i$. A twin law describes the symmetry of a domain pair: $\mathbf{F}_i \cap \mathbf{F}_k$ is the group that simultaneously leaves each domain invariant and $g_{ik}^*(\mathbf{F}_i \cap \mathbf{F}_k)$ a set of elements that interexchanges the two domains. It follows from the preceding paragraph that a non-ferroelastic domain pair (S_i, S_k) has a twin law of the form $\mathbf{J}_{ik} = \mathbf{F} + g_{ik}\mathbf{F}$, a completely (since $\mathbf{F}_i = \mathbf{F}_k$) transposable (since there exists an element g_{ik}^*) twin law. Since the two domains have identical strain tensors, Fam(**F**) = Fam($\mathbf{J}_{ik}$).

The equivalence of two twin laws is defined as follows: two twin laws $\mathbf{J}_{ik} = \mathbf{F} + g_{ik}\mathbf{F}$ and $\mathbf{J}'_{ik} = \mathbf{F}' + g_{ik}^*\mathbf{F}'$ are said to be equivalent and belong to the same class of twin laws if there exists an Euclidean transformation that simultaneously transforms $\mathbf{J}_{ik}$ into $\mathbf{J}'_{ik}$ and **F** into **F**′. For non-magnetic domains, where the symmetries of the domains are non-magnetic groups, the twin laws of non-ferroelastic domain pairs are classified into 43 classes (Litvin & Janovec, 2004); for magnetic domains, where the symmetry of the domains is magnetic groups, there are 309 classes. One representative twin law from each of these 309 classes is listed in Table 1. These classes of magnetic non-ferroelastic twin laws are a subset, that subset where both $\mathbf{J}_{ik}$ and **F** belong to the same crystal family, of the 380 classes of magnetic completely transposable twin laws $\mathbf{J}_{ik} = \mathbf{F} + g_{ik}\mathbf{F}$ tabulated by Litvin *et al.* (1995).

3. Tensor distinction

It is the transformational properties of the components of property tensors under the elements of the non-ferroelastic twin laws $\mathbf{J}_{ik} = \mathbf{F} + g_{ik}\mathbf{F}$ that give rise to the characteristic tensor distinction of a non-ferroelastic domain pair (S_i, S_k). Since the symmetry group of domain state S_i is **F**, the form of the property tensor in S_i is invariant under the group **F**. As the element g_{ik} transforms the domain state S_i into the domain state S_k, it is the transformational properties of the components of the property tensors under this element g_{ik} that determines the characteristic tensor distinction of the non-ferroelastic domain pair. Consider a non-ferroelastic domain pair (S_i, S_k), the matrix form of the relationship between the forms of a tensor $\boldsymbol{T}$ in the two domains is

$$\mathrm{T}(k)_p = \sum_{q=1}^{m} \mathrm{D}(g_{ik})_{p,q}\mathrm{T}(i)_q, \tag{2}$$

where $\mathrm{T}(i)_q$, $q = 1, 2, \ldots, m$, are the components of the form of the tensor $\boldsymbol{T}$ in domain S_i and $\mathrm{T}(k)_p$, $p = 1, 2, \ldots, m$, the components in domain S_k. We assume we work in a Cartesian coordinate system and the tensor is of rank r. The components of the property tensor $\boldsymbol{T}$ are then $\mathrm{T}(k)_p = \mathrm{T}(k)_{ab\ldots r}$ and $\mathrm{T}(i)_q = \mathrm{T}(i)_{a'b'\ldots r'}$, where each of the indices $a, b, \ldots, r$ and $a', b', \ldots, r'$ takes the three values x, y and z. The values of the components of the matrix $\mathrm{D}(g_{ik})_{p,q}$ are then given by

$$\begin{aligned}\mathrm{D}(g_{ik})_{p,q} &= \mathrm{D}(g_{ik})_{abc\ldots r,a'b'c'\ldots r'}\\ &= \theta(g_{ik})\mathrm{V}(g_{ik})_{aa'}\mathrm{V}(g_{ik})_{bb'}\ldots\mathrm{V}(g_{ik})_{rr'},\end{aligned} \tag{3}$$

where V is the three-dimensional polar vector representation of the rotation group and $\theta(g_{ik})$ equals +1 or −1 depending on the transformation g_{ik} and the transformational properties of the property tensor $\boldsymbol{T}$ under spatial and time inversion (Birss, 1964).

If the matrix form of $\mathrm{D}(g_{ik})_{p,q}$ is diagonal, $\mathrm{D}(g_{ik})_{p,q} = \mathrm{D}(g_{ik})_{p,p}\delta_{p,q}$, and $\mathrm{D}(g_{ik})_{p,p} = \pm 1$, then the components $\mathrm{T}(k)_p = \pm\mathrm{T}(i)_p$. If $\mathrm{D}(g_{ik})_{p,p} = +1$, $\mathrm{T}(k)_p = +\mathrm{T}(i)_p$ and the pth component of the physical property tensor $\boldsymbol{T}$ is the same in the two domains. If $\mathrm{D}(g_{ik})_{p,p} = -1$, $\mathrm{T}(k)_p = -\mathrm{T}(i)_p$ and the pth component of the physical property tensor $\boldsymbol{T}$ distinguishes between the domains solely by a change in sign of the component. A sufficient condition that the matrix $\mathrm{D}(g_{ik})_{p,q} = \mathrm{D}(g_{ik})_{p,p}\delta_{p,q}$ and $\mathrm{D}(g_{ik})_{p,p} = \pm 1$ is that one has chosen the element g_{ik} of the twin law and a coordinate system in which to work where the matrix form of the three-dimensional polar vector representation $\mathrm{V}(g_{ik})$, of equation (3), is diagonal.

In Table 1, in all classes of magnetic non-ferroelastic twin laws, except in the 11 cubic classes where g_{ik} has been chosen as 2_{xy}, m_{xy}, $2'_{xy}$ and m'_{xy}, the vector representation $\mathrm{V}(g_{ik})$ is diagonal. For example, for the twin law $\mathbf{4}'_\mathbf{z}\mathbf{m}_\mathbf{x}\mathbf{m}'_\mathbf{xy} = \mathbf{4}'_\mathbf{z} + m_x\mathbf{4}'_\mathbf{z}$:

$$\mathrm{V}(m_x)_{n,n'} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix}.$$

For the 11 exceptional cubic cases, one can choose a new coordinate system, *e.g.* rotate the old coordinate system by 45° about the z axis. In this new coordinate system, $g_{ik} = 2_x$, m_x, $2'_x$, and m'_x respectively, and $\mathrm{V}(g_{ik})$ is again diagonal. Consequently, for every magnetic non-ferroelastic domain pair, there exists a coordinate system in which physical property tensors distinguish between the two domains in a simple

Table 1
309 classes of magnetic non-ferroelastic twin laws $\mathbf{J}_{ik} = \mathbf{F} + g_{ik}\mathbf{F}$.

The sequential numbering from 1 to 32 on the left denotes the magnetic superfamily (Opechowski, 1986) to which the magnetic point group $\mathbf{J}_{ik}$ belongs.

	$\mathbf{J}_{ik}$	$\mathbf{F}$	g_{ik}
1	$11'$	1	$1'$
2	$\bar{1}$	1	$\bar{1}$
	$\bar{1}1'$	$\bar{1}$	$1'$
	$\bar{1}1'$	$11'$	$\bar{1}$
	$\bar{1}1'$	$\bar{1}'$	$1'$
	$\bar{1}'$	1	$\bar{1}'$
3	$2_z1'$	2_z	$1'$
	$2_z1'$	$2'_z$	$1'$
4	m_z1'	m_z	$1'$
	m_z1'	m'_z	$1'$
5	$2_z/m_z$	2_z	$\bar{1}$
	$2_z/m_z$	m_z	$\bar{1}$
	$2_z/m_z1'$	$2_z/m_z$	$1'$
	$2_z/m_z1'$	$2_z1'$	$\bar{1}$
	$2_z/m_z1'$	m_z1'	$\bar{1}$
	$2_z/m_z1'$	$2_z/m'_z$	$1'$
	$2_z/m_z1'$	$2'_z/m_z$	$1'$
	$2_z/m_z1'$	$2'_z/m'_z$	$1'$
	$2'_z/m_z$	m_z	$\bar{1}'$
	$2'_z/m_z$	$2'_z$	$\bar{1}'$
	$2_z/m'_z$	2_z	$\bar{1}'$
	$2_z/m'_z$	m'_z	$\bar{1}'$
	$2'_z/m'_z$	$2'_z$	$\bar{1}$
	$2'_z/m'_z$	m'_z	$\bar{1}$
6	$2_x2_y2_z1'$	$2_x2_y2_z$	$1'$
	$2_x2_y2_z1'$	$2'_x2'_y2_z$	$1'$
7	$m_xm_y2_z1'$	$m_xm_y2_z$	$1'$
	$m_xm_y2_z1'$	$m'_xm_y2'_z$	$1'$
	$m_xm_y2_z1'$	$m'_xm'_y2_z$	$1'$
8	$m_xm_ym_z$	$2_x2_y2_z$	$\bar{1}$
	$m_xm_ym_z$	$m_xm_y2_z$	$\bar{1}$
	$m_xm_ym_z1'$	$m_xm_ym_z$	$1'$
	$m_xm_ym_z1'$	$m_xm_y2_z1'$	$\bar{1}$
	$m_xm_ym_z1'$	$2_x2_y2_z1'$	$\bar{1}$
	$m_xm_ym_z1'$	$m'_xm_ym_z$	$1'$
	$m_xm_ym_z1'$	$m'_xm'_ym_z$	$1'$
	$m_xm_ym_z1'$	$m'_xm'_ym'_z$	$1'$
	$m'_xm_ym_z$	$m_xm_y2_z$	$\bar{1}'$
	$m'_xm_ym_z$	$m'_xm_y2'_z$	$\bar{1}'$
	$m'_xm_ym_z$	$2_x2'_y2'_z$	$\bar{1}'$
	$m'_xm'_ym_z$	$m'_xm'_y2_z$	$\bar{1}$
	$m'_xm'_ym_z$	$m'_xm_y2'_z$	$\bar{1}$
	$m'_xm'_ym_z$	$2'_x2'_y2_z$	$\bar{1}$
	$m'_xm'_ym'_z$	$2_x2_y2_z$	$\bar{1}'$
	$m'_xm'_ym'_z$	$m'_xm'_y2_z$	$\bar{1}'$
9	$4_z1'$	4_z	$1'$
	$4_z1'$	$4'_z$	$1'$
10	$\bar{4}_z1'$	$\bar{4}_z$	$1'$
	$\bar{4}_z1'$	$\bar{4}'_z$	$1'$
11	$4_z/m_z$	4_z	$\bar{1}$
	$4_z/m_z$	$\bar{4}_z$	$\bar{1}$
	$4_z/m_z1'$	$4_z/m_z$	$1'$
	$4_z/m_z1'$	$4_z1'$	$\bar{1}$
	$4_z/m_z1'$	$\bar{4}_z1'$	$\bar{1}$
	$4_z/m_z1'$	$4'_z/m_z$	$1'$
	$4_z/m_z1'$	$4_z/m'_z$	$1'$
	$4_z/m_z1'$	$4'_z/m'_z$	$1'$
	$4'_z/m_z$	$4'_z$	$\bar{1}$
	$4'_z/m_z$	$\bar{4}'_z$	$\bar{1}$
	$4_z/m'_z$	4_z	$\bar{1}'$
	$4_z/m'_z$	$\bar{4}'_z$	$\bar{1}'$
	$4'_z/m'_z$	$\bar{4}_z$	$\bar{1}'$
	$4'_z/m'_z$	$4'_z$	$\bar{1}'$

Table 1 (continued)

	$\mathbf{J}_{ik}$	$\mathbf{F}$	g_{ik}
12	$4_z2_x2_{xy}$	4_z	2_x
	$4_z2_x2_{xy}1'$	$4_z2_x2_{xy}$	$1'$
	$4_z2_x2_{xy}1'$	$4_z1'$	2_x
	$4_z2_x2_{xy}1'$	$4'_z2_x2'_{xy}$	$1'$
	$4_z2_x2_{xy}1'$	$4_z2'_x2'_{xy}$	$1'$
	$4'_z2_x2'_{xy}$	$4'_z$	2_x
	$4_z2'_x2'_{xy}$	4_z	$2'_x$
13	$4_zm_xm_{xy}$	4_z	m_x
	$4_zm_xm_{xy}1'$	$4_zm_xm_{xy}$	$1'$
	$4_zm_xm_{xy}1'$	$4_z1'$	m_x
	$4_zm_xm_{xy}1'$	$4'_zm_xm'_{xy}$	$1'$
	$4_zm_xm_{xy}1'$	$4_zm'_xm'_{xy}$	$1'$
	$4'_zm_xm'_{xy}$	$4'_z$	m_x
	$4'_zm'_xm'_{xy}$	4_z	m'_x
14	$\bar{4}_z2_xm_{xy}$	$\bar{4}_z$	2_x
	$\bar{4}_z2_xm_{xy}1'$	$\bar{4}_z2_xm_{xy}$	$1'$
	$\bar{4}_z2_xm_{xy}1'$	$\bar{4}_z\ 1'$	2_x
	$\bar{4}_z2_xm_{xy}1'$	$\bar{4}'_z2_xm'_{xy}$	$1'$
	$\bar{4}_z2_xm_{xy}1'$	$\bar{4}'_z2'_xm_{xy}$	$1'$
	$\bar{4}_z2_xm_{xy}1'$	$\bar{4}_z2'_xm'_{xy}$	$1'$
	$\bar{4}'_z2_xm'_{xy}$	$\bar{4}'_z$	2_x
	$\bar{4}'_z2'_xm_{xy}$	$\bar{4}'_z$	$2'_x$
	$\bar{4}_z2'_xm'_{xy}$	$\bar{4}_z$	$2'_x$
15	$4_z/m_zm_xm_{xy}$	$4_z/m_z$	2_x
	$4_z/m_zm_xm_{xy}$	$4_z2_x2_{xy}$	$\bar{1}$
	$4_z/m_zm_xm_{xy}$	$4_zm_xm_{xy}$	$\bar{1}$
	$4_z/m_zm_xm_{xy}$	$\bar{4}_z2_xm_{xy}$	$\bar{1}$
	$4_z/m_zm_xm_{xy}1'$	$4_z/m_zm_xm_{xy}$	$1'$
	$4_z/m_zm_xm_{xy}1'$	$4_z/m_z1'$	2_x
	$4_z/m_zm_xm_{xy}1'$	$4_z2_x2_{xy}1'$	$\bar{1}$
	$4_z/m_zm_xm_{xy}1'$	$4_zm_xm_{xy}1'$	$\bar{1}$
	$4_z/m_zm_xm_{xy}1'$	$\bar{4}_z2_xm_{xy}1'$	$\bar{1}$
	$4_z/m_zm_xm_{xy}1'$	$4'_z/m_zm_xm'_{xy}$	$1'$
	$4_z/m_zm_xm_{xy}1'$	$4_z/m_zm'_xm'_{xy}$	$1'$
	$4_z/m_zm_xm_{xy}1'$	$4_z/m'_zm'_xm'_{xy}$	$1'$
	$4_z/m_zm_xm_{xy}1'$	$4_z/m'_zm_xm_{xy}$	$1'$
	$4_z/m_zm_xm_{xy}1'$	$4'_z/m'_zm'_xm_{xy}$	$1'$
	$4_z/m_zm'_xm'_{xy}$	$4_z/m_z$	$2'_x$
	$4_z/m_zm'_xm'_{xy}$	$4_z2'_x2'_{xy}$	$\bar{1}$
	$4_z/m_zm'_xm'_{xy}$	$4_zm'_xm'_{xy}$	$\bar{1}$
	$4_z/m_zm'_xm'_{xy}$	$\bar{4}_z2'_xm'_{xy}$	$\bar{1}$
	$4'_z/m_zm_xm'_{xy}$	$4'_z/m_z$	2_x
	$4'_z/m_zm_xm'_{xy}$	$4'_z2_x2'_{xy}$	$\bar{1}$
	$4'_z/m_zm_xm'_{xy}$	$4'_zm_xm'_{xy}$	$\bar{1}$
	$4'_z/m_zm_xm'_{xy}$	$\bar{4}'_z2_xm'_{xy}$	$\bar{1}$
	$4'_z/m_zm_xm'_{xy}$	$\bar{4}'_zm_x2'_{xy}$	$\bar{1}$
	$4_z/m'_zm'_xm'_{xy}$	$4_z2_x2_{xy}$	$\bar{1}'$
	$4_z/m'_zm'_xm'_{xy}$	$4_z/m'_z$	2_x
	$4_z/m'_zm'_xm'_{xy}$	$4_zm'_xm'_{xy}$	$\bar{1}'$
	$4_z/m'_zm'_xm'_{xy}$	$\bar{4}'_z2_xm'_{xy}$	$\bar{1}'$
	$4_z/m'_zm_xm_{xy}$	$4_zm_xm_{xy}$	$\bar{1}'$
	$4_z/m'_zm_xm_{xy}$	$4_z/m'_z$	$2'_x$
	$4_z/m'_zm_xm_{xy}$	$4_z2'_x2'_{xy}$	$\bar{1}'$
	$4_z/m'_zm_xm_{xy}$	$\bar{4}'_z2'_xm_{xy}$	$\bar{1}'$
	$4'_z/m'_zm'_xm_{xy}$	$\bar{4}_z2_xm_{xy}$	$\bar{1}'$
	$4'_z/m'_zm'_xm_{xy}$	$4'_z/m'_z$	2_x
	$4'_z/m'_zm'_xm_{xy}$	$4'_z2_x2'_{xy}$	$\bar{1}'$
	$4'_z/m'_zm'_xm_{xy}$	$4'_zm'_xm_{xy}$	$\bar{1}'$
	$4'_z/m'_zm'_xm_{xy}$	$\bar{4}_z\ m'_x2'_{xy}$	$\bar{1}'$
16	$3_z1'$	3_z	$1'$
17	$\bar{3}_z$	3_z	$\bar{1}$
	$\bar{3}_z1'$	$\bar{3}_z$	$1'$
	$\bar{3}_z1'$	$3_z1'$	$\bar{1}$
	$\bar{3}_z1'$	$\bar{3}'_z$	$1'$
	$\bar{3}_z$	3_z	$\bar{1}'$
18	3_z2_x	3_z	2_x
	$3_z2_x1'$	3_z2_x	$1'$
	$3_z2_x1'$	$3_z1'$	2_x

Table 1 (continued)

	$\mathbf{J}_{ik}$	$\mathbf{F}$	g_{ik}
	$3_z2_x1'$	$3_z2'_x$	$1'$
	$3_z2'_x$	3_z	$2'_x$
19	3_zm_x	3_z	m_x
	$3_zm_x1'$	3_zm_x	$1'$
	$3_zm_x1'$	$3_z1'$	m_x
	$3_zm_x1'$	$3_zm'_x$	$1'$
	$3_zm'_x$	3_z	m'_x
20	$\bar{3}_zm_x$	$\bar{3}_z$	2_x
	$\bar{3}_zm_x$	3_zm_x	$\bar{1}$
	$\bar{3}_zm_x$	3_z2_x	$\bar{1}$
	$\bar{3}_zm_x1'$	$\bar{3}_zm_x$	$1'$
	$\bar{3}_zm_x1'$	$\bar{3}_z1'$	2_x
	$\bar{3}_zm_x1'$	$3_zm_x1'$	$\bar{1}$
	$\bar{3}_zm_x1'$	$3_z2_x1'$	$\bar{1}$
	$\bar{3}_zm_x1'$	$\bar{3}_zm'_x$	$1'$
	$\bar{3}_zm_x1'$	$\bar{3}'_zm_x$	$1'$
	$\bar{3}_zm_x1'$	$\bar{3}'_zm'_x$	$1'$
	$\bar{3}_zm'_x$	$\bar{3}_z$	$2'_x$
	$\bar{3}_zm'_x$	$3_zm'_x$	$\bar{1}$
	$\bar{3}_zm'_x$	$3_z2'_x$	$\bar{1}$
	$\bar{3}'_zm_x$	3_zm_x	$\bar{1}'$
	$\bar{3}'_zm_x$	$\bar{3}'_z$	m_x
	$\bar{3}'_zm_x$	$3_z2'_x$	$\bar{1}'$
	$\bar{3}'_zm'_x$	3_z2_x	$\bar{1}'$
	$\bar{3}'_zm'_x$	$\bar{3}'_z$	2_x
	$\bar{3}'_zm'_x$	$3_zm'_x$	$\bar{1}'$
21	6_z	3_z	2_z
	$6_z1'$	6_z	$1'$
	$6_z1'$	$3_z1'$	2_z
	$6_z1'$	$6'_z$	$1'$
	$6'_z$	3_z	$2'_z$
22	$\bar{6}_z$	3_z	m_z
	$\bar{6}_z1'$	$\bar{6}_z$	$1'$
	$\bar{6}_z1'$	$3_z1'$	m_z
	$\bar{6}_z1'$	$\bar{6}'_z$	$1'$
	$\bar{6}'_z$	3_z	m'_z
23	$6_z/m_z$	6_z	$\bar{1}$
	$6_z/m_z$	$\bar{6}_z$	$\bar{1}$
	$6_z/m_z$	$\bar{3}_z$	2_z
	$6_z/m_z1'$	$6_z/m_z$	$1'$
	$6_z/m_z1'$	$6_z1'$	$\bar{1}$
	$6_z/m_z1'$	$\bar{6}_z1'$	$\bar{1}$
	$6_z/m_z1'$	$\bar{3}_z1'$	2_z
	$6_z/m_z1'$	$6_z/m'_z$	$1'$
	$6_z/m_z1'$	$6'_z/m_z$	$1'$
	$6_z/m_z1'$	$6'_z/m'_z$	$1'$
	$6_z/m'_z$	6_z	$\bar{1}'$
	$6_z/m'_z$	$\bar{6}'_z$	$\bar{1}'$
	$6_z/m'_z$	$\bar{3}'_z$	2_z
	$6'_z/m_z$	$\bar{6}_z$	$\bar{1}'$
	$6'_z/m_z$	$6'_z$	$\bar{1}'$
	$6'_z/m_z$	$\bar{3}'_z$	$2'_z$
	$6'_z/m'_z$	$\bar{3}_z$	$2'_z$
	$6'_z/m'_z$	$6'_z$	$\bar{1}$
	$6'_z/m'_z$	$\bar{6}'_z$	$\bar{1}$
24	$6_z2_x2_1$	3_z2_x	2_z
	$6_z2_x2_1$	6_z	2_x
	$6_z2_x2_11'$	$6_z2_x2_1$	$1'$
	$6_z2_x2_11'$	$3_z2_x1'$	2_z
	$6_z2_x2_11'$	$6_z1'$	2_x
	$6_z2_x2_11'$	$6'_z2_x2'_1$	$1'$
	$6_z2_x2_11'$	$6_z2'_x2'_1$	$1'$
	$6'_z2_x2'_1$	3_z2_x	$2'_z$
	$6'_z2_x2'_1$	$3_z2'_x$	$2'_z$
	$6'_z2_x2'_1$	$6'_z$	2_x
	$6_z2'_x2'_1$	6_z	$2'_x$
	$6_z2'_x2'_1$	$3_z2'_x$	2_z
25	$6_zm_xm_1$	3_zm_x	2_z
	$6_zm_xm_1$	6_z	m_x
	$6_zm_xm_11'$	$6_zm_xm_1$	$1'$
	$6_zm_xm_11'$	$3_zm_x1'$	2_z
	$6_zm_xm_11'$	$6_z1'$	m_x
	$6_zm_xm_11'$	$6'_zm_xm'_1$	$1'$
	$6_zm_xm_11'$	$6_zm'_xm'_1$	$1'$
	$6'_zm_xm'_1$	3_zm_x	$2'_z$
	$6'_zm_xm'_1$	$3_zm'_x$	$2'_z$
	$6'_zm_xm'_1$	$6'_z$	m_x
	$6_zm'_xm'_1$	6_z	m'_x
	$6_zm'_xm'_1$	$3_zm'_x$	2_z
26	$\bar{6}_zm_x2_1$	3_z2_1	m_z
	$\bar{6}_zm_x2_1$	3_zm_x	m_z
	$\bar{6}_zm_x2_1$	$\bar{6}_z$	m_x
	$\bar{6}_zm_x2_11'$	$\bar{6}_zm_x2_1$	$1'$
	$\bar{6}_zm_x2_11'$	$3_z2_11'$	m_z
	$\bar{6}_zm_x2_11'$	$3_zm_x1'$	m_z
	$\bar{6}_zm_x2_11'$	$\bar{6}_z1'$	m_x
	$\bar{6}_zm_x2_11'$	$\bar{6}'_zm'_x2_1$	$1'$
	$\bar{6}_zm_x2_11'$	$\bar{6}'_zm_x2'_1$	$1'$
	$\bar{6}_zm_x2_11'$	$\bar{6}_zm'_x2'_1$	$1'$
	$\bar{6}'_zm_x2'_1$	3_zm_x	m'_z
	$\bar{6}'_zm_x2'_1$	$3_z2'_1$	m'_z
	$\bar{6}'_zm_x2'_1$	$\bar{6}'_z$	m_x
	$\bar{6}'_zm'_x2_1$	3_z2_1	m'_z
	$\bar{6}'_zm'_z2_1$	$3_zm'_x$	m'_z
	$\bar{6}'_zm'_x2_1$	$\bar{6}'_z$	m'_x
	$\bar{6}_zm'_x2'_1$	$\bar{6}_z$	m'_x
	$\bar{6}_zm'_x2'_1$	$3_zm'_x$	m_z
	$\bar{6}_zm'_x2'_1$	$3_z2'_1$	m_z
27	$6_z/m_zm_xm_1$	$\bar{3}_zm_x$	2_z
	$6_z/m_zm_xm_1$	$6_z/m_z$	2_x
	$6_z/m_zm_xm_1$	$6_z2_x2_1$	$\bar{1}$
	$6_z/m_zm_xm_1$	$6_zm_xm_1$	$\bar{1}$
	$6_z/m_zm_xm_1$	$\bar{6}_zm_x2_1$	$\bar{1}$
	$6_z/m_zm_xm_11'$	$6_z/m_zm_xm_1$	$1'$
	$6_z/m_zm_xm_11'$	$\bar{3}_zm_x1'$	2_z
	$6_z/m_zm_xm_11'$	$6_z/m_z1'$	2_x
	$6_z/m_zm_xm_11'$	$6_z2_x2_11'$	$\bar{1}$
	$6_z/m_zm_xm_11'$	$6_zm_xm_11'$	$\bar{1}$
	$6_z/m_zm_xm_11'$	$\bar{6}_zm_x2_11'$	$\bar{1}$
	$6_z/m_zm_xm_11'$	$6'_z/m'_zm_xm'_1$	$1'$
	$6_z/m_zm_xm_11'$	$6_z/m_zm'_xm'_1$	$1'$
	$6_z/m_zm_xm_11'$	$6_z/m'_zm'_xm'_1$	$1'$
	$6_z/m_zm_xm_11'$	$6_z/m'_zm_xm_1$	$1'$
	$6_z/m_zm_xm_11'$	$6'_z/m_zm_xm'_1$	$1'$
	$6'_z/m'_zm_xm'_1$	$\bar{3}_zm_x$	$2'_z$
	$6'_z/m'_zm_xm'_1$	$\bar{3}_zm'_1$	$2'_z$
	$6'_z/m'_zm_xm'_1$	$6'_z/m'_z$	m_x
	$6'_z/m'_zm_xm'_1$	$6'_z2_x2'_1$	$\bar{1}$
	$6'_z/m'_zm_xm'_1$	$6'_zm_xm'_1$	$\bar{1}$
	$6'_z/m'_zm_xm'_1$	$\bar{6}'_zm_x2'_1$	$\bar{1}$
	$6'_z/m'_zm_xm'_1$	$\bar{6}'_z2_xm'_1$	$\bar{1}$
	$6_z/m_zm'_xm'_1$	$6_z/m_z$	$2'_x$
	$6_z/m_zm'_xm'_1$	$\bar{3}_zm'_x$	2_z
	$6_z/m_zm'_xm'_1$	$6_z2'_x2'_1$	$\bar{1}$
	$6_z/m_zm'_xm'_1$	$6_zm'_xm'_1$	$\bar{1}$
	$6_z/m_zm'_xm'_1$	$\bar{6}_zm'_x2'_1$	$\bar{1}$
	$6_z/m'_zm'_xm'_1$	$6_z2_x2_1$	$\bar{1}'$
	$6_z/m'_zm'_xm'_1$	$\bar{3}'_zm'_x$	2_z
	$6_z/m'_zm'_xm'_1$	$6_z/m'_z$	2_x
	$6_z/m'_zm'_xm'_1$	$6_zm'_xm'_1$	$\bar{1}'$
	$6_z/m'_zm'_xm'_1$	$\bar{6}'_zm'_x2_1$	$\bar{1}'$
	$6_z/m'_zm_xm_1$	$6_zm_xm_1$	$\bar{1}'$
	$6_z/m'_zm_xm_1$	$\bar{3}'_zm_x$	2_z
	$6_z/m'_zm_xm_1$	$6_z/m'_z$	$2'_x$
	$6_z/m'_zm_xm_1$	$6_z2'_x2'_1$	$\bar{1}'$
	$6_z/m'_zm_xm_1$	$\bar{6}'_zm_x2'_1$	$\bar{1}'$

Table 1 (continued)

	$\mathbf{J}_{ik}$	$\mathbf{F}$	g_{ik}
	$6'_z/m_z m_x m'_1$	$\bar{6}_z m_x 2_1$	$\bar{1}'$
	$6'_z/m_z m_x m'_1$	$\bar{3}'_z m_x$	$2'_z$
	$6'_z/m_z m_x m'_1$	$\bar{3}'_z m'_x$	$2'_z$
	$6'_z/m_z m_x m'_1$	$6'_z/m_z$	$2'_x$
	$6'_z/m_z m_x m'_1$	$6'_z 2'_x 2_1$	$\bar{1}'$
	$6'_z/m_z m_x m'_1$	$\bar{6}'_z m_x m'_1$	$\bar{1}'$
	$6'_z/m_z m_x m'_1$	$\bar{6}_z 2'_x m'_1$	$\bar{1}'$
28	$2_z 3_{xyz} 1'$	$2_z 3_{xyz}$	$1'$
29	$m_z \bar{3}_{xyz}$	$2_z 3_{xyz}$	$\bar{1}$
	$m_z \bar{3}_{xyz} 1'$	$m_z \bar{3}_{xyz}$	$1'$
	$m_z \bar{3}_{xyz} 1'$	$2_z 3_{xyz} 1'$	$\bar{1}$
	$m_z \bar{3}_{xyz} 1'$	$m'_z \bar{3}'_{xyz}$	$1'$
	$m'_z \bar{3}'_{xyz}$	$2_z 3_{xyz}$	$\bar{1}'$
30	$4_z 3_{xyz} 2_{xy}$	$2_z 3_{xyz}$	2_{xy}
	$4_z 3_{xyz} 2_{xy} 1'$	$4_z 3_{xyz} 2_{xy}$	$1'$
	$4_z 3_{xyz} 2_{xy} 1'$	$2_z 3_{xyz} 1'$	2_{xy}
	$4_z 3_{xyz} 2_{xy} 1'$	$4'_z 3_{xyz} 2'_{xy}$	$1'$
	$4'_z 3_{xyz} 2'_{xy}$	$2_z 3_{xyz}$	$2'_{xy}$
31	$\bar{4}_z 3_{xyz} m_{xy}$	$2_z 3_{xyz}$	m_{xy}
	$\bar{4}_z 3_{xyz} m_{xy} 1'$	$\bar{4}_z 3_{xyz} m_{xy}$	$1'$
	$\bar{4}_z 3_{xyz} m_{xy} 1'$	$2_z 3_{xyz} 1'$	m_{xy}
	$\bar{4}_z 3_{xyz} m_{xy} 1'$	$\bar{4}'_z 3_{xyz} m'_{xy}$	$1'$
	$\bar{4}'_z 3_{xyz} m'_{xy}$	$2_z 3_{xyz}$	m'_{xy}
32	$m_z \bar{3}_{xyz} m_{xy}$	$m_z \bar{3}_{xyz}$	2_{xy}
	$m_z \bar{3}_{xyz} m_{xy}$	$\bar{4}_z 3_{xyz} m_{xy}$	$\bar{1}$
	$m_z \bar{3}_{xyz} m_{xy}$	$4_z 3_{xyz} 2_{xy}$	$\bar{1}$
	$m_z \bar{3}_{xyz} m_{xy} 1'$	$m_z \bar{3}_{xyz} m_{xy}$	$1'$
	$m_z \bar{3}_{xyz} m_{xy} 1'$	$m_z \bar{3}_{xyz}\ 1'$	2_{xy}
	$m_z \bar{3}_{xyz} m_{xy} 1'$	$\bar{4}_z 3_{xyz} m_{xy} 1'$	$\bar{1}$
	$m_z \bar{3}_{xyz} m_{xy} 1'$	$4_z 3_{xyz} 2_{xy} 1'$	$\bar{1}$
	$m_z \bar{3}_{xyz} m_{xy} 1'$	$m_z \bar{3}_{xyz} m'_{xy}$	$1'$
	$m_z \bar{3}_{xyz} m_{xy} 1'$	$m'_z \bar{3}'_{xyz} m_{xy}$	$1'$
	$m_z \bar{3}_{xyz} m_{xy}\ 1'$	$m'_z \bar{3}'_{xyz} m'_{xy}$	$1'$
	$m_z \bar{3}_{xyz} m'_{xy}$	$m_z \bar{3}_{xyz}$	$2'_{xy}$
	$m_z \bar{3}_{xyz} m'_{xy}$	$4'_z 3_{xyz} 2'_{xy}$	$\bar{1}$
	$m_z \bar{3}_{xyz} m'_{xy}$	$\bar{4}'_z 3_{xyz} m'_{xy}$	$\bar{1}$
	$m'_z \bar{3}'_{xyz} m'_{xy}$	$4_z 3_{xyz} 2_{xy}$	$\bar{1}'$
	$m'_z \bar{3}'_{xyz} m'_{xy}$	$m'_z \bar{3}'_{xyz}$	2_{xy}
	$m'_z \bar{3}'_{xyz} m'_{xy}$	$\bar{4}'_z 3_{xyz} m'_{xy}$	$\bar{1}'$
	$m'_z \bar{3}'_{xyz} m_{xy}$	$\bar{4}_z 3_{xyz} m_{xy}$	$\bar{1}'$
	$m'_z \bar{3}'_{xyz} m_{xy}$	$m'_z \bar{3}'_{xyz}$	$2'_{xy}$
	$m'_z \bar{3}'_{xyz} m_{xy}$	$4'_z 3_{xyz} 2'_{xy}$	$\bar{1}'$

manner: each component of the property tensor is either the same in the two domains or differs only in its sign.

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